A High Performance Implementation of the Escape Time Algorithm to generate the Mandelbrot Set

Parallel and high-performance computing

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Scientific Background

The Mandelbrot set is the set of complex numbers c for which the function $f_c(z) = z^2 + c$ does not diverge when iterated from $z = 0$.

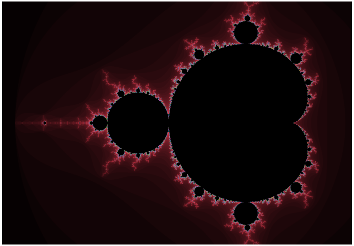
If we denote the Mandelbrot set by M, then by repeatedly applying the quadratic map

$$\begin{cases} z_0 = 0 \\ z_{n+1} = z_n + c, \end{cases}$$

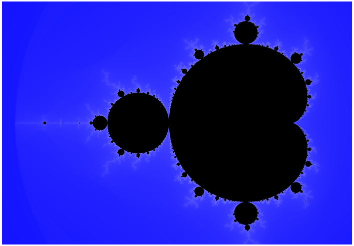
$\forall$ complex number c, we have $c \in M \iff \limsup_{n\to\infty} |z_{n+1}| \leq 2$.

Generating Mandelbrot set images

- Sample complex numbers and iterate this function for each point c. If the function goes to infinity, the point belongs to the set.
- Each sampled point can be mapped on a $2D$ plane:
  - Treat its real and imaginary parts as image coordinates $(x + yi)$
  - Color this pixel according to how quickly $z_n^2 + c$ diverges



(a) Red-shaded



(b) Smooth, blue-shaded

Figure 1: Mandelbrot set drawings generated with our palettes.

The Escape Time Algorithm

Algorithm 1 Escape Time algorithm

for each pixel (Px, Py) on the screen **do**
  $x_0 \leftarrow$ scaled x coordinate of pixel
  $y_0 \leftarrow$ scaled y coordinate of pixel
  $x \leftarrow 0.0$
  $y \leftarrow 0.0$
  $iteration \leftarrow 0$
  while $(x \times x + y \times y < 2 \times 2$ AND $iteration < max\_iteration)$ **do**
    $x_{temp} \leftarrow x \times x - y \times y + x_0$
    $y \leftarrow 2 \times x \times y + y_0$
    $x \leftarrow x_{temp}$
    $iteration \leftarrow iteration + 1$
  $color \leftarrow palette[iteration]$
  $plot(Px, Py, color)$

Where $z = x + iy$, $c = x_0 + iy_0$, $x = Re(z^2 + c)$ and $y = Im(z^2 + c)$.

Embarrassingly parallel problem: each pixel independent of any other.

Implementations

Implementations overview

Three main classes of implementations are developed:

- Serial versions.
- GPU versions using the `CUDA` parallel computing platform.
- Hybrid versions using both `CUDA` and `MPI` (including `MPI-IO`).

The application is coded in `C`.

Code fully debugged and profiled using `gdb` and `gprof`.
`Valgrind` used to make sure no memory leaks are possible.

Serial Implementations (1)

- **naive**.
- **opti1:** Exploit symmetry with respect to the x axis.
- **opti2:** Skip calculations for the points lying within the cardioid or in the period-2 bulb ($\approx 34\%$ of the total area).
- **opti3:** Finer-grained optimizations: reduce to 3 multiplications per loop (minimum) & use fast array indexing in plotting.

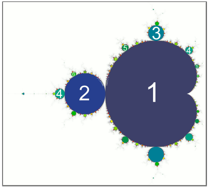
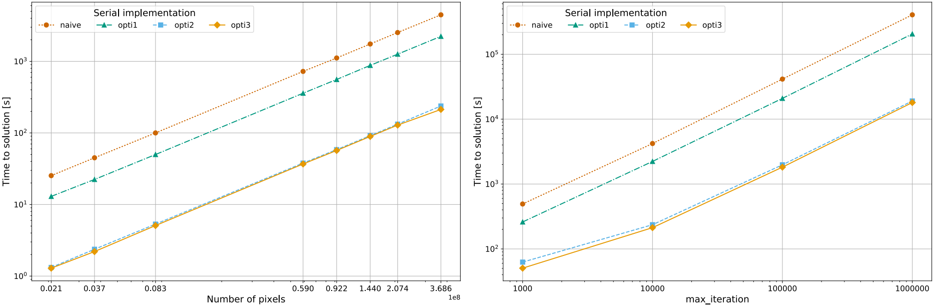


Figure 2: Periods of hyperbolic components.

Optimization level	Execution time [s]
`-O0`	422.91
`-O1`	230.99
`-O2`	208.96
`-O3`	208.49
`-O3 -ftree-vectorize`	205.41

Table 1: Execution times of `opti3` ($368.64M$ pixels, max_iteration=$10,000$) as a function of the optimization level.

Serial Implementations (2)



(a) **(b)**

Figure 3: Time to solution for the different serial implementations with respect to (a) the number of pixels (max_iteration=10,000) and (b) max_iteration (368,640,000 pixels).

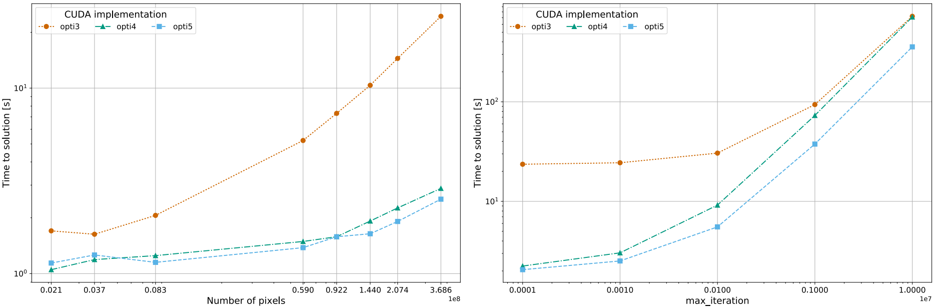
Going Parallel

CUDA Implementations (1)

- **opti3:** Parallelize escape time evaluation only.
  - Colors computed serially
  - 3 matrices sent on the PCIe bus
- **opti4:** All workload carried out by the GPU.
  - Only exchange RGB values of the bottom part on the PCIe bus
  - Use *Constant memory* to store constant values on the GPU
  - Code split into serial (`.c`) & parallel (`.cu`) parts; needs C wrapper
- **opti5:** Single-precision implementation of `opti4`.
  - Maximize instruction throughput
  - Cannot distinguish between double- and single-precision images

Application profiled with `nvprof`: compute-bound.

CUDA Implementations (2)



(a) **(b)**

Figure 4: Time to solution for the CUDA implementations with respect to (a) the number of pixels (max_iteration=10,000) and (b) max_iteration (368,640,000 pixels) with block size 512×1.

CUDA *block size* tuning for `opti5`

Block size:

- hyperparameter leading to performance improvements, increasing *occupancy* $\left(\frac{active\ warps}{maximum\ active\ warps}\right)$
- It depends on both the problem and the specifics of the GPU

Block size	Occupancy [%]	
	2560x1440	25600x14400
32x1	18.05	18.81
64x1	25.41	32.65
128x1	31.82	51.52
256x1	**36.15**	**58.80**
512x1	28.76	52.37
1024x1	8.39	25.50
8x8	19.04	20.12
16x16	26.91	25.93
32x32	5.91	7.74

Block size	Execution time [s]	
	2560x1440	25600x14400
32x1	2.53	79.61
64x1	2.29	50.32
128x1	2.10	37.52
256x1	**2.04**	**35.91**
512x1	2.17	37.69
1024x1	5.65	90.60
8x8	4.09	237.72
16x16	2.45	97.12
32x32	7.48	270.24

Table 2: Occupancy and execution time for two image sizes as the block size varies for max_iteration$=10,000$. Evaluated using `nvprof`.

Hybrid Implementations (1)

Distribute the computations over different CPUs and GPUs with `MPI`.

- Image split to the 2 processors in a node, each uses 1 GPU.
  - Non-blocking, point-to-point communications where rank 0-processor receives the image parts out-of-order [1]
  - Parallel I/O
- Image split to the 2 processors in a node, each uses 2 GPUs. Highest performance: all the CUDA functions are asynchronous [2]
  - Non-blocking, point-to-point communications where rank 0-processor receives the image parts out-of-order [1]
  - Parallel I/O

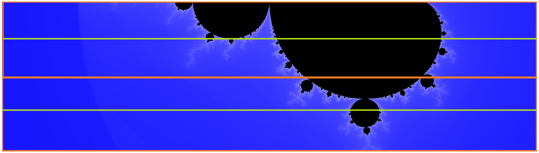
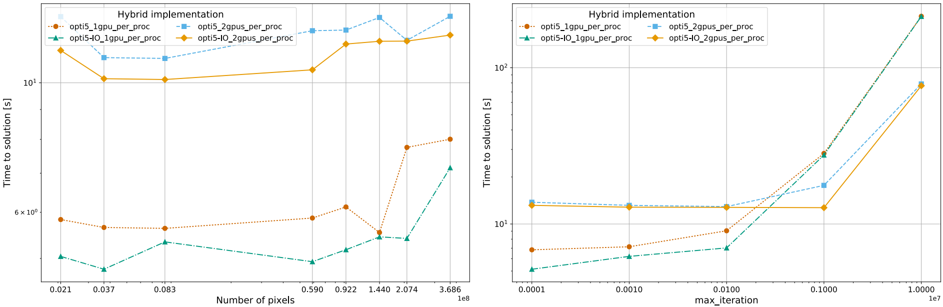


Figure 5: Image split to CPUs (orange) and GPUs (orange or green).

[1] No `MPI_Barrier()` used.

[2] No `__syncthreads()` used.

Hybrid Implementations (2)

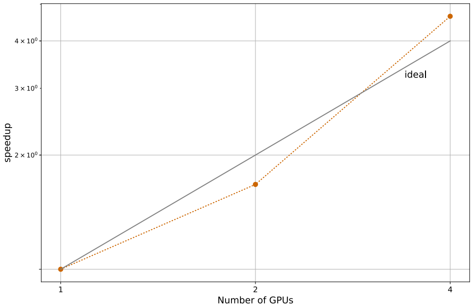


(a) (b)

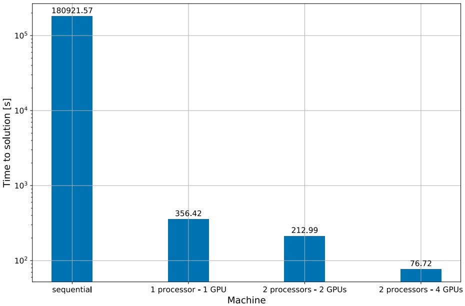
Figure 6: Time to solution for the hybrid implementations with respect to (a) the number of pixels (max_iteration=10,000) and (b) max_iteration (368,640,000 pixels) with block size 256×1.

Results

Strong Scaling



(a)



(b)

Figure 7: Strong scaling as (a) the speedup for an increasing number of GPUs (via `MPI`) and (b) the reduced time to solution of the same problem ($368.64M$-pixel image, max_iteration=10×10^6) for the different versions.

Weak Scaling

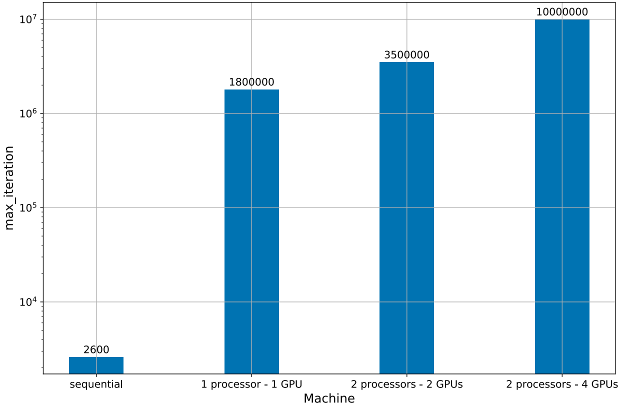


Figure 8: Weak scaling as a function of the problem size that can be solved in the same amount of time (76 s) by the different versions.

Conclusion

Conclusion & Future Work

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- Implement and optimize `serial`, `CUDA` and `CUDA+MPI` applications
- Gain experience with `CUDA` and `MPI` in a supercomputer
- Acquire familiarity with profiling tools
- Investigate different technologies
  - Non-blocking communications and parallel I/O in `MPI`
  - Asynchronous functions in `CUDA`

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- Experiment with 2D grids
- Investigate the impact of external libraries
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Thank you