# Matrix Completion in the Unit Hypercube via Structured Matrix Factorization

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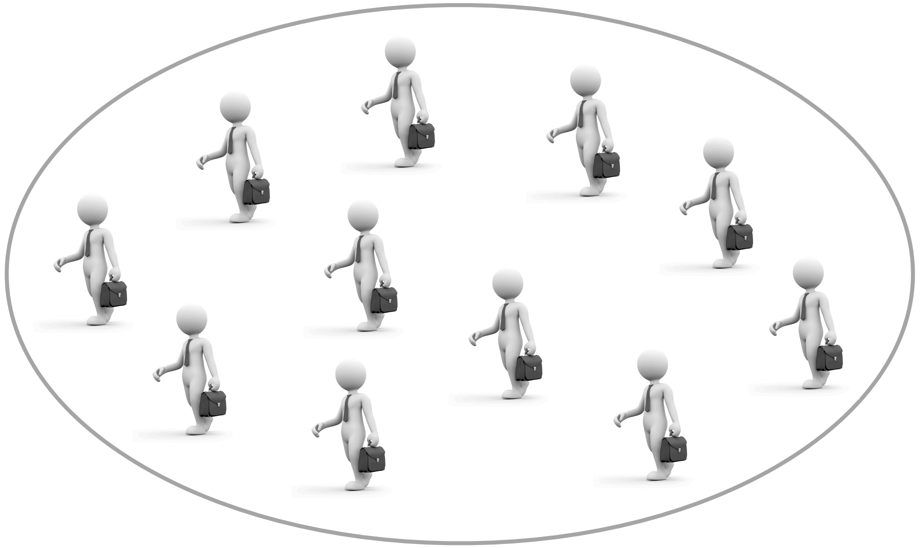

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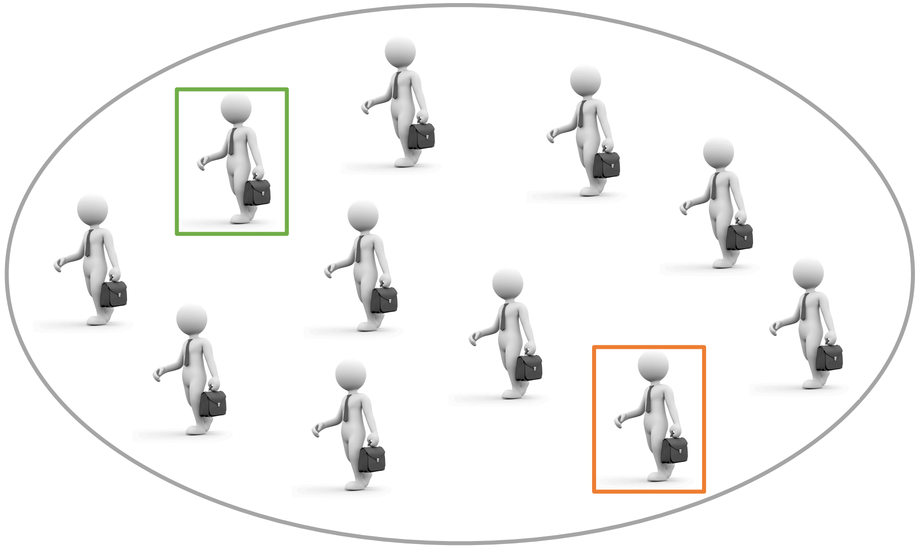

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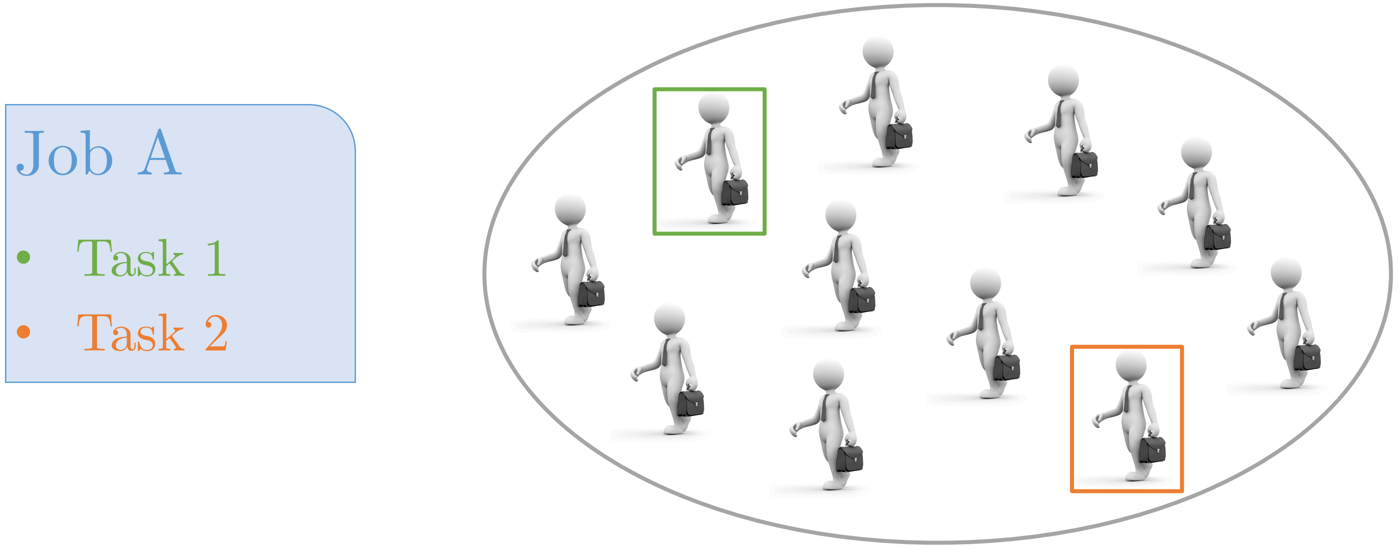


- Approach: Predict the efficiency of employees in each task!

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    - *expertise matrix factorization* (EMF): exploits the [0, 1] boundary constraint
    - *survival matrix factorization* (SMF): probabilistic model of employee efficiency

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- Propose 2 novel structured matrix factorization models that leverage our knowledge of the environment
  - *expertise matrix factorization* (EMF): exploits the [0, 1] boundary constraint
  - *survival matrix factorization* (SMF): probabilistic model of employee efficiency

- Validate the effectiveness of our models on 3 real-world datasets with values bounded in the [0, 1] interval

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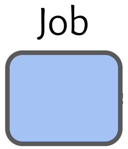

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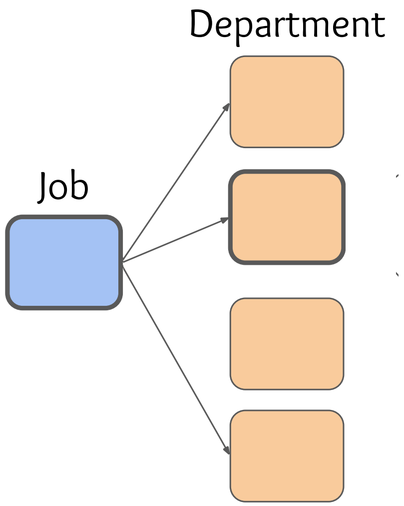


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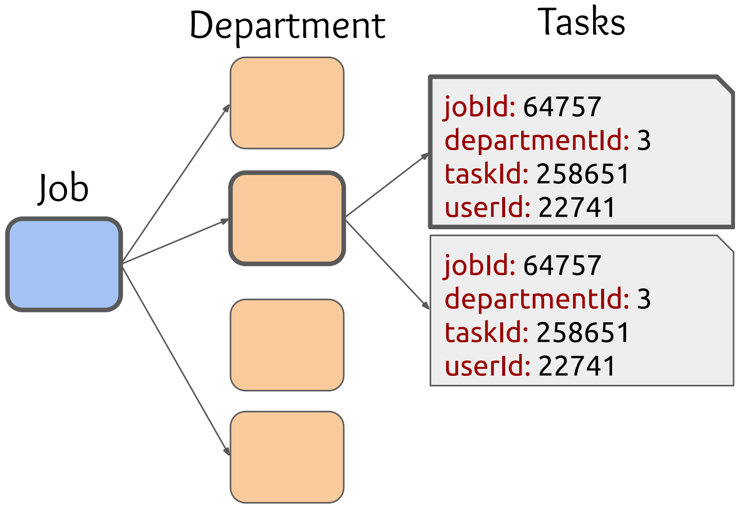


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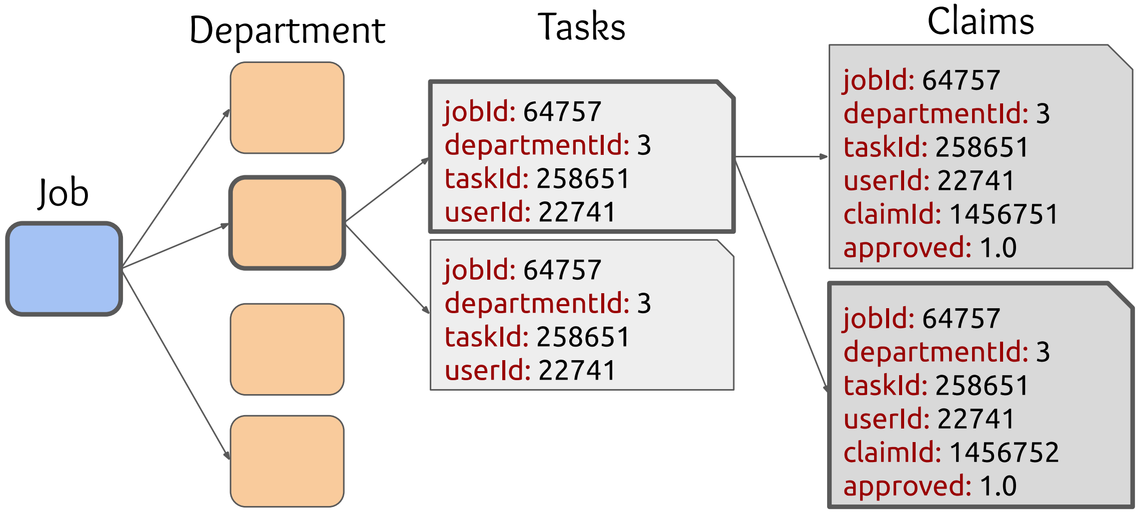


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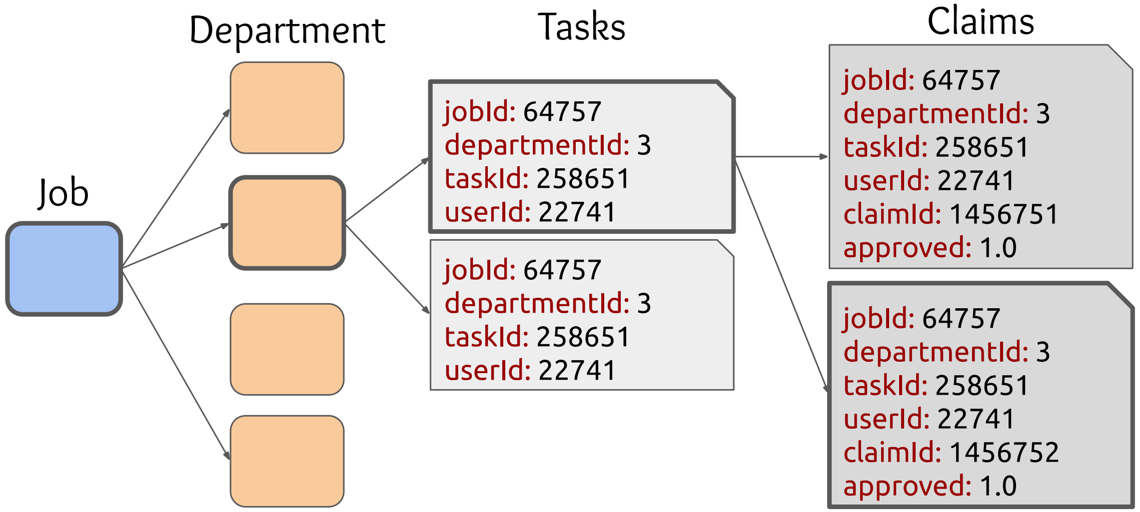


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  - Each department is led by a *manager* who divides the work in their department into *tasks*
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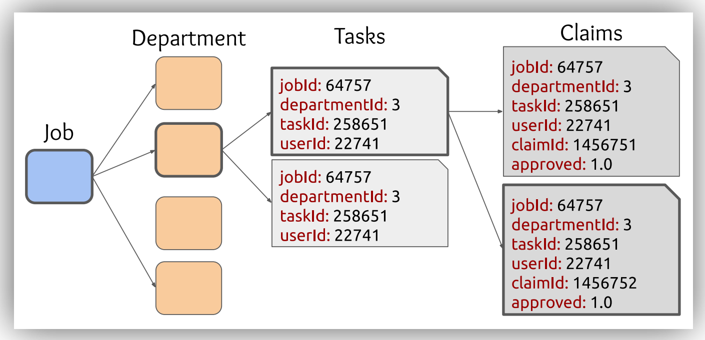
  - Managers assess the quality of claims and decide to approve them or not

# Problem Formulation (2/2)



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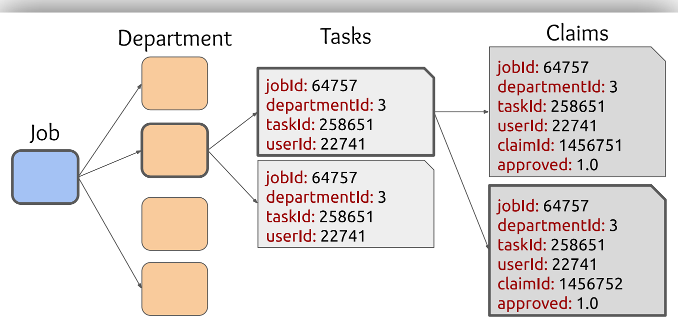
- Employee's competency: ratio of accepted claims
  - Rejected claim ⇒ performance loss for the organization



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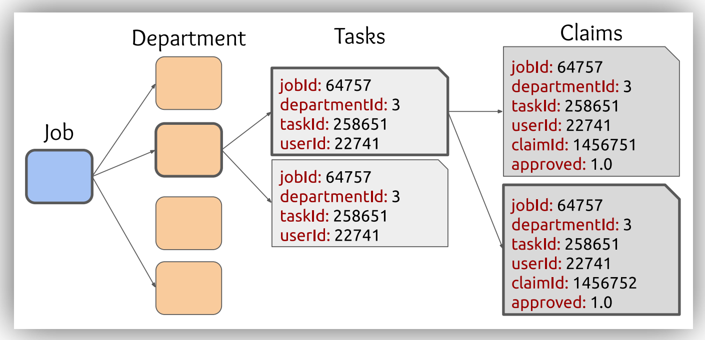


- Employee's competency: ratio of accepted claims
  - Rejected claim $\Rightarrow$ performance loss for the organization

- We define the efficiency of employee $d$ in department $n$ as:

$$x_{dn} = \frac{\sum_{i=1}^{N_{dn}} a_{dn}^{(i)}}{N_{dn}} \in [0,1]$$

  - $N_{dn}$ : total number of claims by employee $d$ in department $n$
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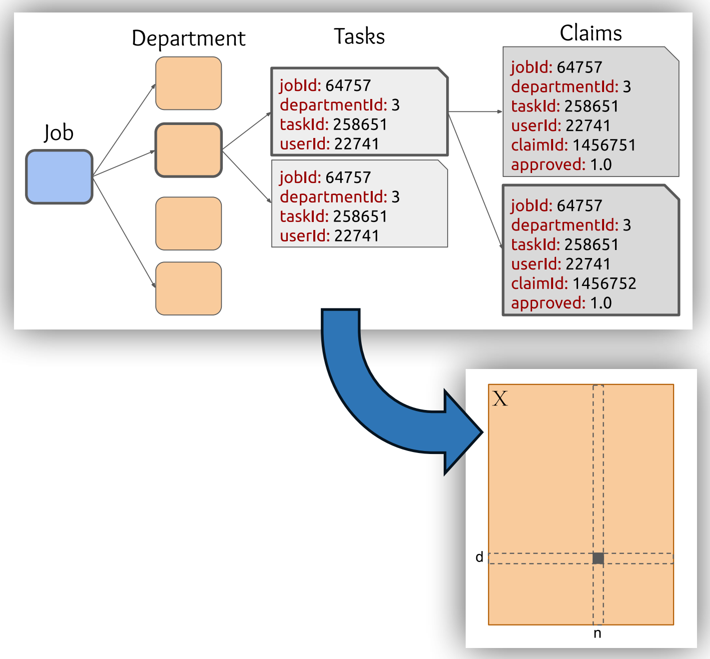


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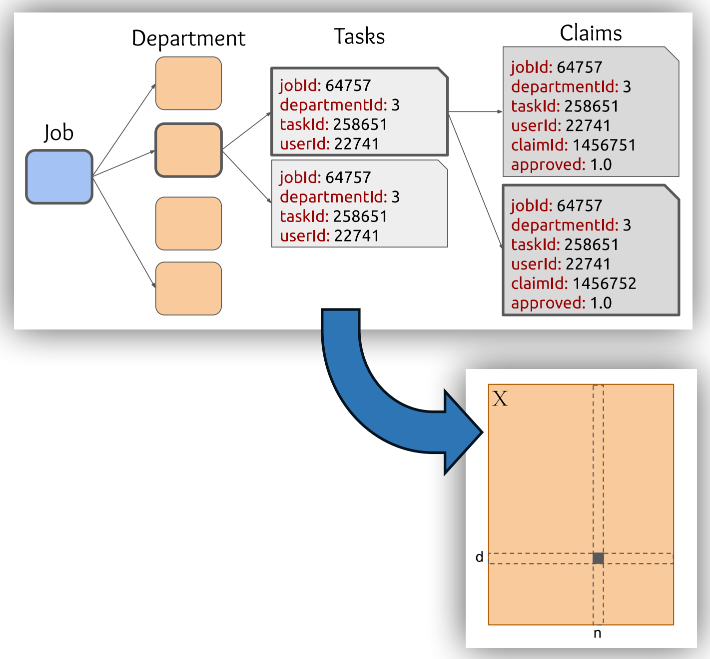


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- Most employees work in a few departments ⇒ efficiency matrix $\mathbf{X}$ is sparse

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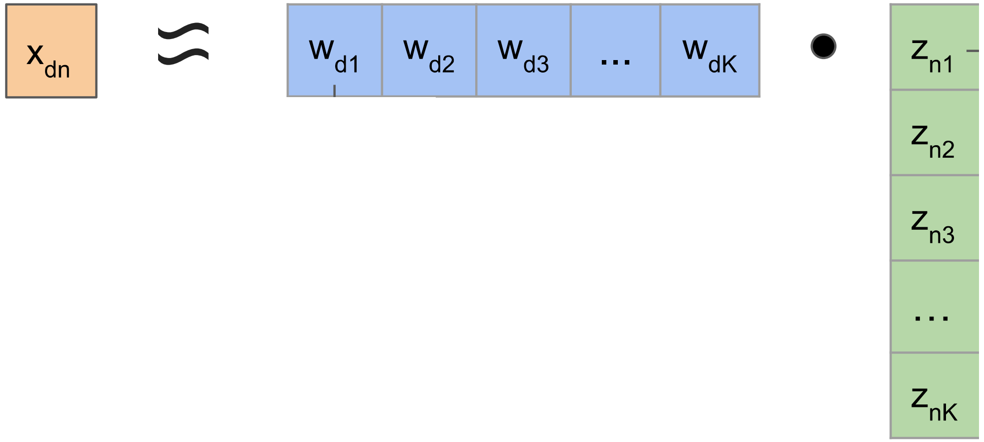

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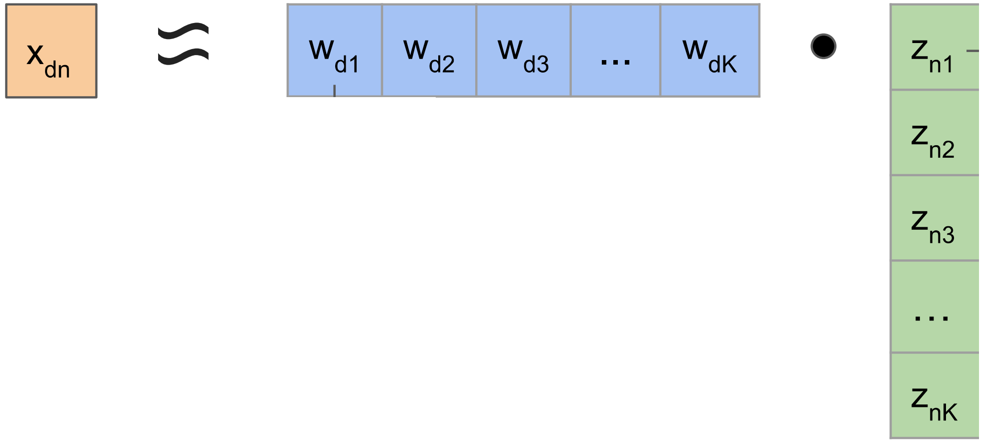


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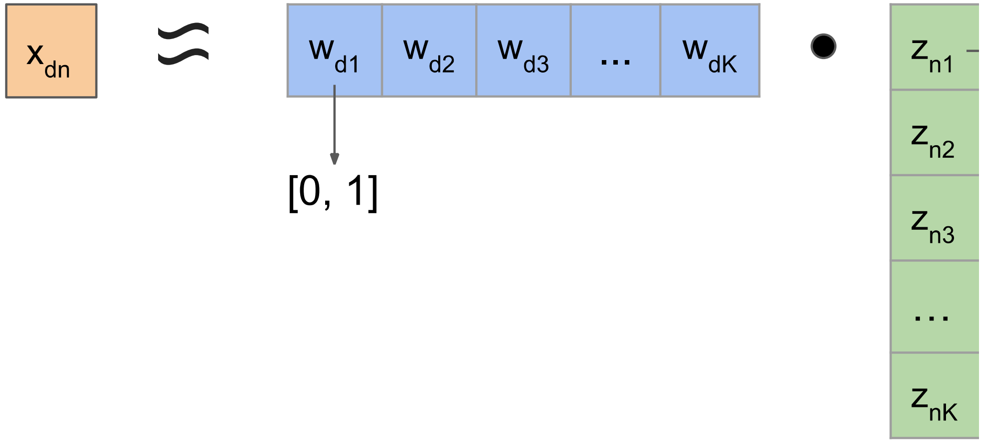


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  - Employees' latent factors (skills): from 0 (no ability) to 1 (proficiency)
  - Departments' latent factors: non-negative and sum to 1
    - each department has a distribution of skills required to complete tasks in it

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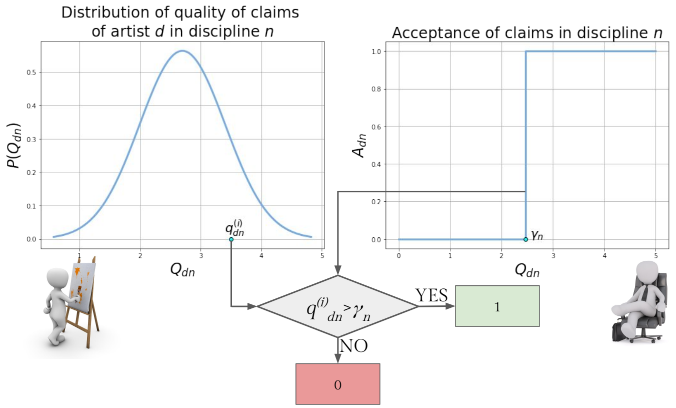

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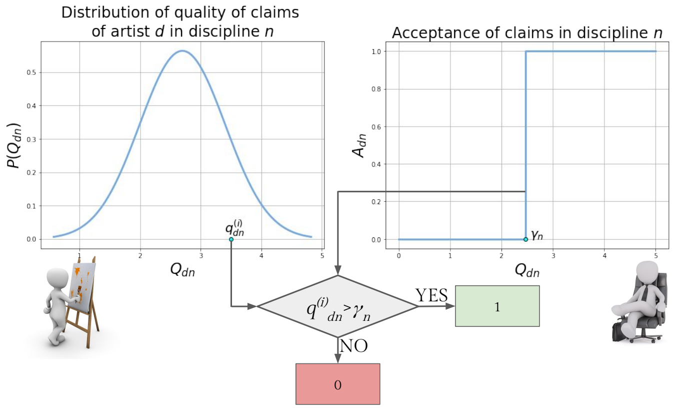


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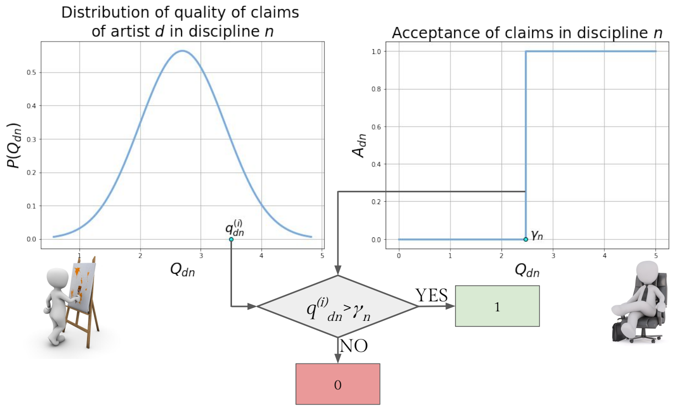


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  - $S_{Q_{dn}}\left(\gamma_n\right)$ is the survival function of $Q_{dn}$ at $\gamma_n$
  - Assume: Gaussian quality distribution $f_{Q_{dn}}$ with variance $\sigma^2$ and mean $\mu_{dn} \approx \mathbf{w}_d^T \cdot \mathbf{z}_n$

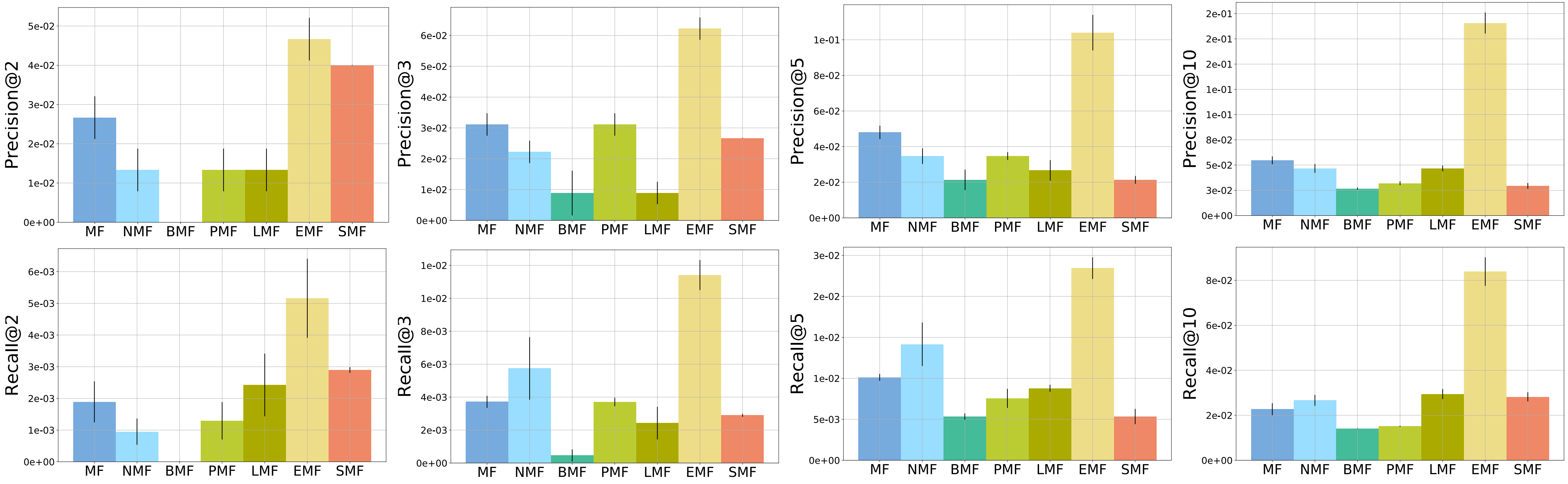
$$x_{dn} \approx \int_{\gamma_n}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{\left(u - \mathbf{w}_d^T \cdot \mathbf{z}_n\right)^2}{2\sigma^2}\right] du$$

# Experiments: Methodology

- Datasets
  - Movie Production [In-house]
  - Over-The-Top [In-house]
  - Click-Through Rate [Public – Outbrain Click Prediction competition]
- Evaluation metrics
  - Precision@$N$
  - Recall@$N$
- Baselines
  - MF: Matrix Factorization
  - NMF: Nonnegative Matrix Factorization
  - BMF: Bounded Matrix Factorization [Kannan et al., 2014]
  - PMF: Probabilistic Matrix Factorization [Mnih and Salakhutdinov, 2008]
  - LMF: Logistic Matrix Factorization [Johnson, 2014]

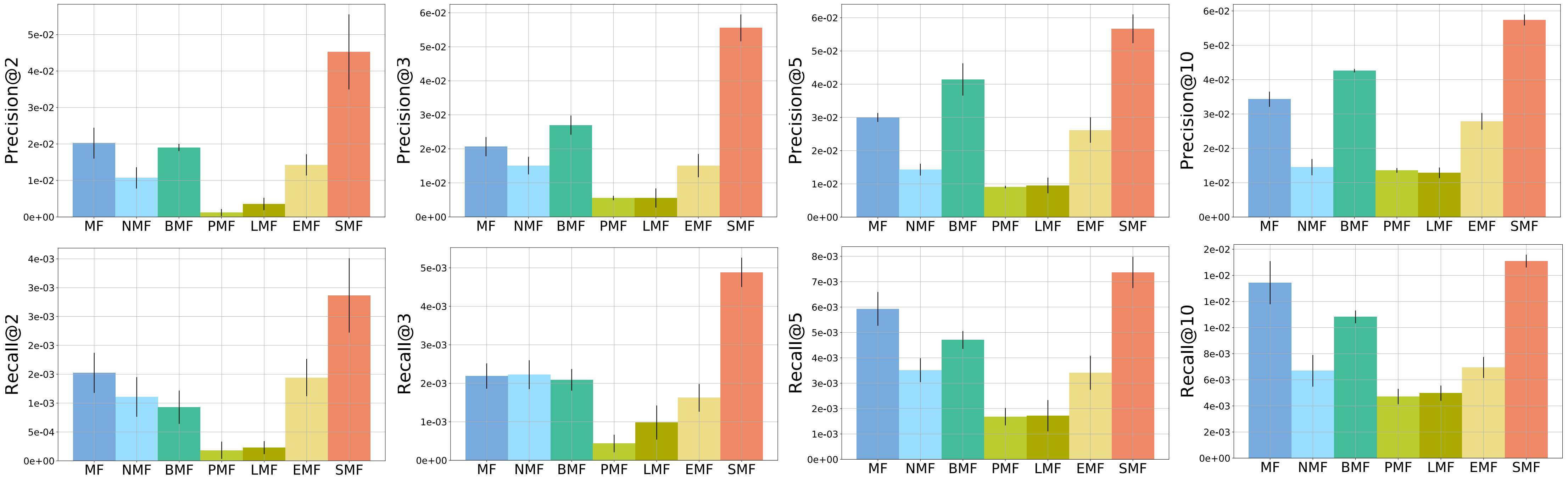
# Experiments: Movie Production Data

- Recommend employees to departments according to their efficiencies
- $\mathbf{X}$ : $312 \times 25$ – $86.85\,\%$ sparse – *very few ratings per user*



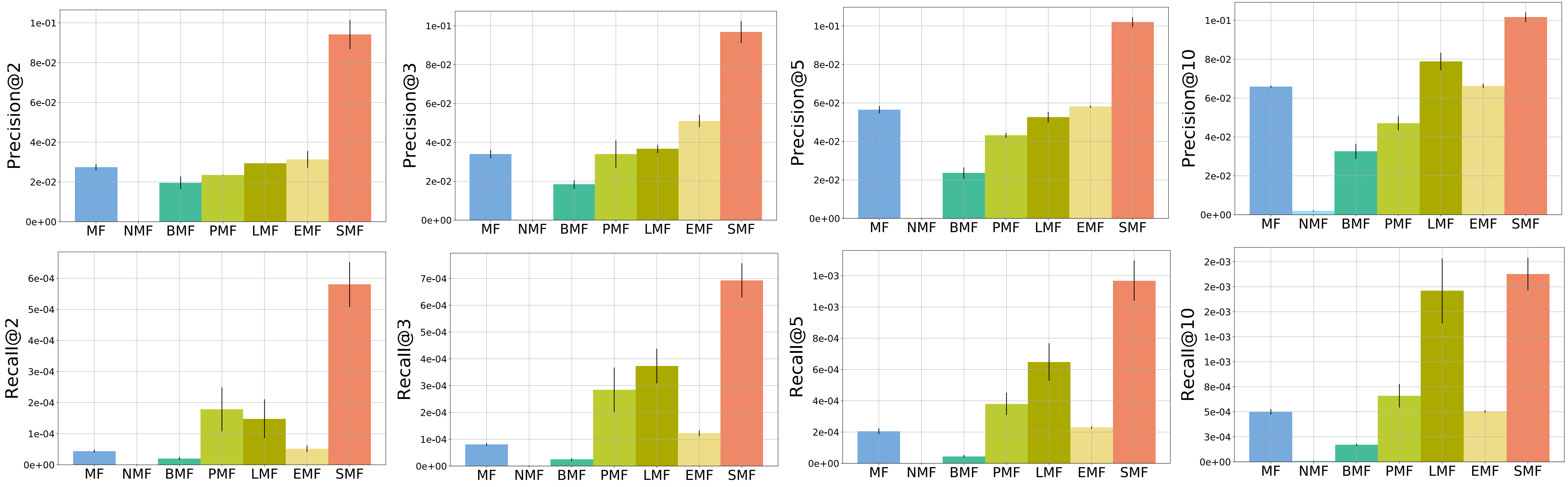
# Experiments: Over-The-Top Data

- Recommend apps to users according to their watching rates
- $\mathbf{X}$ : $934 \times 140$ – $99.91\ \%$ sparse



# Experiments: Click-Through Rate Data

- Recommend website categories to ad placer to maximize click-through rate
- $\mathbf{X} : 15647 \times 85 - 79.99\,\%$ sparse



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  - The hypotheses of EMF match the Movie Production framework, not the others
  - EMF outperforms PMF in a dataset where users have very few ratings (max 7)

- SMF is a general model, outperforming every method in the other datasets
  - SMF underperforms in the Movie Production dataset due to very few available entries rather than due to sparsity (Over-The-Top data is much sparser)

# Conclusion

- EMF and SMF: Two novel structured MF techniques for entries in [0, 1]
  - They give better recommendations than popular MF techniques
  - It is beneficial to explicitly model entries in [0, 1]

- SMF is the simplest in a class of probabilistic models
  - Here, we assumed
    - Normally-distributed quality of work
    - Managers modeled by a single threshold

Poster #663

Experimental Results: https://github.com/e-bug/unit-mf

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